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Performance evaluation of peering-agreements among autonomous systems subject to peer-to-peer traffic

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Abstract

The interconnection of thousands of Autonomous Systems (ASs) makes up the Internet. Each AS shares trade agreements with its neighbors that regulate the costs associated with traffic exchanged on the physical links. These agreements are local, i.e., are settled only between directly connected ASs, but have a global impact by influencing the paths allowed for the routing of network packets and the costs associated with these routes. Indeed, the costs and earnings of interconnected ASs is function of many factors, such as size of the ASs, existing agreements, routing policy, traffic pattern and AS-level topology. In this paper we present an approach that takes these factors into account to assess peering and transit agreements. Here we focus on traffic generated from P2P activities, but the approach is general enough to be applied to different traffic classes. The P2P model we present is based on the use of the generating function, it allows to perform an analytical study of the traffic associated to file-sharing. The proposed P2P model is able to consider large number of peers sharing several resources, spread along different ASs connected through a series of links. We validate the results of our P2P model against one of the most widely used P2P simulator, i.e. PeerSim. Using both the AS-level and P2P model we evaluate how the inter-AS P2P traffic influences the AS network cost and earning.

Keywords: Peer-to-peer Networks, Autonomous Systems, Generating Function

1. Introduction

Studying the relationships between "Autonomous Systems" (ASs) has become an important research issue. Indeed, Internet traffic is generated from applications that are agnostic of the underlying AS topology. This leads to poor usage of network's resources, resulting in an economic damage for ASs. The latter must be studied under realistic traffic assumptions, considering common applications, such as peer-to-peer, Internet 2.0 and social networks.

URL: `http://www.dei.polimi.it/` (Daniele Manini), `http://www.di.unito.it/` (Daniele Manini)

In particular, peer-to-peer (P2P) applications generate huge data flows that nowadays is a major fraction of the Internet traffic [33]. P2P systems can be used in different contexts, such as file sharing (Gnutella [37], KaZaa [38], eDonkey [28], BitTorrent [29]), telephony applications (Skype [39]), and content delivery infrastructures (see [16]). File-sharing is one of the most popular where contents such as multimedia and software are spread over the network. The size of resources ranges from several kilobytes up to some gigabytes. In order to understand the dynamics of such applications, it is interesting to study how shared resources move across the network. Despite the large number of parameters involved, P2P file sharing applications might have different impact on the overall network performance depending on their setting. For instance, a P2P client could be encouraged to seek required contents in peers belonging to the same AS in order to reduce the traffic in the network.

This work exploits the results presented in [22] to study the impact of the commercial relationship among different autonomous systems on the costs of supporting P2P traffic. In particular, in this paper we study the diffusion of a resource in a non-homogeneous environment. Peers are considered being spread among the ASs, each one having its own parameters in terms of resource availability and demand. We propose a probabilistic model that describes the diffusion of a resource taking into account (non-homogeneous) resource popularity and peers behavior. The model is extended to also consider multi-resource scenario where each peer can hold more resource types.

The main goal of this paper is to provide the system administrator the tools to minimize its cost related to the traffic produced by the users of its network. In particular, we want to show how this can be achieved by either trying to settle different peering agreements with other ASs, or by applying intelligent routing schemes that can provide a cheaper use of its resources. Our contribution is twofold: first we present a methodology to account for cost and rewards in complex AS topologies under a given traffic pattern, then we propose an analytical procedure to describe such a pattern in the case of P2P file sharing networks.

The reminder of the paper is organized as follows. After a brief discussion on some related work (Section 2), we describe how we modeled topology and relationships between ASs (Section 3). The resources diffusion in a P2P overlay network is considered in Sections 4, 5, 6. We validate the proposed model by simulation in Section 7. Finally, we apply the proposed technique in a complex scenario (Section 8).

2. Background and Related Work

In this section, we briefly describe the Internet's AS-level, explaining peering and transit agreements between ASs and inter-AS routing protocols. We describe previous attempts in literature to model or optimize AS relationships and routing. We introduce the P2P paradigm and review some of the existing resource diffusion models.

2.1. The Internet AS-level

The term Autonomous System informally identifies a set of routers under the same technical administration, that share common metrics to route packets within the AS, while use an inter-AS protocol for forward network messages to other. An AS appears to its neighbors having a single coherent internal routing plan, announcing routes that are reachable through it.

A more rigorous definition is given in RFC 1930 [10] that defines an AS as a group of IP prefixes that share a unique and clearly defined routing policy. Here ”prefix” is referred to a CIDR block, a group of one or more classful networks (A, B or C networks).

In Internet each AS is uniquely identified by an Autonomous System Number (ASN), a 32 bit integer as specified in RFC 4893 [35]. ASN are assigned by the Internet Assigned Numbers Authority (IANA) to Regional Internet Registry (RIRS). RIRSs are responsible for specific geographic areas and in turn assign ASNs to organizations that make request.

The protocol used nowadays to exchange routing and reachability information is the Border Gateway Protocol (BGP) [25, 26, 34, 4]. The current version is BGP-4. BGP-4 is defined as an exterior gateway protocol used for intra-AS routing, in contrast to the interior gateway protocols, like Routing Information Protocol (RIP) [11] or Intermediate System To Intermediate System (IS-IS) [24]), used within the same AS. Each AS runs some BGP-4 gateways that discover routes exchanging reachability information with other gateway. Each one announces the destinations (i.e other ASs), that are reachable through it.

BGP is a *Path Vector Protocol*, so gateways exchange full AS-paths, according to their routing policies. These determine what are the best paths to reach a specific destination. A common parameter is the length of AS-path [31], preferring shorter over longer ones, but the AS administration can specify more complex policies.

2.2. Traffic agreements: peering and transit

According to the BGP Topological Model (cf. RFC 1655 [31], Section 2), a direct connection between two AS is both a physical connection (i.e., a shared network formed at least from one border gateway for each AS) and a BGP session running on the border routers (gateways). A connection is demanding in terms of economical resources (hardware, maintenance, technical administration), so commercial agreements are settled between directly connected ASs. An AS could be connected to more ASs and have a specific agreement with each one. In this case the AS is called *multihomed*. Otherwise, a *stub* is an AS that is connected to only one AS.

Among the existing established methods to exchange Internet traffic between directly connected networks, the most used are **peering** and **transit**. In peering two networks do not charge any fees to each other, while a transit agreement occurs when an AS pays to another some fees to reach other parts of Internet. In this case the traffic travels across the seller AS and is forwarded to the next hop to destination.

Peering and transit agreements can influence the way messages are routed on the AS topology because BGP policies used from a network are based on economical and commercial considerations [26]. Indeed, is convenient for an AS to announce routes for other networks it peers only to its customers. Usually traffic between two peering partners is not forwarded. In the same way it is not allowed to route traffic from peering partners to seller ASs and vice-versa. The reasons behind these policies are simple to understand: if an AS announces these kind of routes, it would be providing free transit over its network for its peers or buy transit from another network and giving it away freely to a peer. For sake of clarity we illustrate some allowed paths for the topology depicted in Figure 2:

- AS_8 can see all the networks because networks AS_5, AS_6, AS_7 buy transit from it.
- AS_1 can see AS_2 and its customers directly, but not AS_3 through network AS_2.
- Traffic from AS_4 to AS_7 is routed by AS_6, but not through AS_5.
- AS_4 can see Network B through its peer AS_5, but not via its transit customer AS_2.

Peering and transit agreements also induce a hierarchy on AS topology. At the top there are **Tier 1** networks (AS_8 in the example in Figure 2), that sell transit traffic to all its partners and can reach every other AS without pay any settlements or buy transit traffic. A **Tier 2** network (the other ASs in Figure 2) buys transit traffic but has also some peering agreements. Finally, the term **Tier 3** is sometimes used to refer to networks that only purchase transit traffic.

2.3. Modeling and improving intra-AS routing

Many research works have investigated the interaction between p2p protocols and ASs or Internet Service Providers (ISPs). Often the proposed techniques aim to minimize the inter-domain traffic while maintaining an acceptable quality of service. On the other side, few research efforts has been directed towards the modeling of the complex network of commercial relationships between ASs. Model the AS-level is crucial in order to understand how the network-aware techniques and application protocols influence costs and rewards of ASs.

Indeed, practically there is always a tradeoff between network awareness and protocol performance. Some authors argue [27] [42] that AS-aware techniques lead to performance degradation in many practical conditions. In [27] a measuring study demonstrates as a BitTorrent client in practice has few peers in its neighborhood that belong to the same AS, so using a locality-based approach has a too high impact on protocol performance.

Exploiting locality is one of the most diffused strategies for cost minimization. Several proposals that use this approach are summarized in [6]. The key point is to identify the p2p traffic, via the ports it uses or packets inspection, and redirect to the same network or throttle the bandwidth of the heaviest users. Redirection often requires the implementation of tracker oracles that select neighbor peers from the same AS as proposed in [1], [40], [5] and [17]. The work presented in [17] proposes an ISP-friendly protocol to control the cross-ISP traffic for reducing congestion and operating costs. This protocol is based on the idea that a peer downloads resources first among the peers belonging to the same AS, such as the policy presented in our work in Section 5.2. First authors develop a mathematical model to support the efficiency of the idea. Then they implement it on BitTorrent clients in order to evaluate via experiments the cross-ISP traffic reduction and, moreover, the average file downloading time. Although our model does not consider time issues, this limit can be easily relaxed by associating time/bandwith costs to resource transfers. The ISP-friendly protocol is implemented at application level with clients exploiting information provided by their ISP, whereas in our work the searching policies should be performed at AS level. We think that ASs can plan their searching policies analyzing the traffic and cost evaluation derived by our generating function based model.

Exploring the effectiveness of network-aware techniques requires a model that takes into account commercial relationships between ASs, e.g., peering and transit agreements. For example, it could be interesting to explore the consequences of an egoistic AS that reroutes all the p2p traffic towards its customer instead of its provider links. Many works in literature related to the AS-topology investigate the AS-graph formation and evolution [36] [18] [9]. These works start from economic principles and try to capture the salient features of the provider and customer selection process exploring the AS network grow. All these topology evolution models are useful to understand properties of the graph, but don't describe the interaction between this latter and the application level protocols.

In [42] a model for three of the common used strategies for network awareness is proposed. The main limitation of this work is that it is useful only for the described strategies, and it is not clear what is the effort to generalize for more generic ones. The work presented in [6] explores the possibility to optimize the path trading technique between several ASs. When use path trading an AS defers from using hot potato routing and in turn it expects its neighbors do the same. In [42] is shown that the path trading optimization problem is NP-hard, but the authors propose a pseudo polynomial algorithm based on some heuristics. In [19] is presented a more detailed and realistic network model with three classes of ISPs: content, transit, and eye- ball, but only simple scenarios are explored. In [2] an analysis of peering and transit agreements is proposed. The study aims to provide guidelines for solving disputes between ISPs and for establishing regulatory protocols, but also in this case it is limited only to two ISPs.

The main limitation of the previously cited models is that none of them explores the interaction between ASs and application layer protocols. Moreover, these models are often limited to simple scenarios or to a fixed number of strategies. In this work we propose a model for the AS-level that takes into account peering and transit agreements, complex network and is able to compute AS cost and reward distributions.

2.4. P2P models

For what concerns traffic modeling, and in particular P2P systems, there are several works related with our proposal. The work presented in [30] shows a fluid model for the BitTorrent P2P application, and it is able to study steady-state performance measures, such as the number of peers that have a resource and remain in the system to allow its diffusion. In our work we consider the transient behavior instead, by using an embedded process where time is not considered explicitly.

The simulation techniques proposed in [13] investigate the diffusion of a file in a e-Donkey system, as a function of several parameters such as sharing probability and requests arrival rate. The analytical model developed in [15] is based on biological epidemics. In particular, it is used to predict the diffusion of single files in a P2P network, whereas we focus on the diffusion of single or distinct resources among different ASs.

The probabilistic model presented in [23] is inspired by the study of file swarming in BitTorrent like systems. The measurement-based technique utilized in [32] provides static, topological, and dynamic analysis of the P2P Gnutella environment. The dynamic analysis allows to study the variations in terms of popularity of individual files, and in terms of the number of available files at individual peers. We also compute the resource diffusion, but we focus on the traffic among ASs.

One of the models studied in [41] considers how a document is spread to the requesting peers into a bit-torrent environment. The model proposed in [3] describes P2P dynamics through a set of second order fluid equations, that allows to derive results related to the resource distribution among peers. The models developed in [7, 21] are aimed at studying the transfer time distribution of a resource: their goal is to characterize the time required to diffuse a resource from a set of peers holding it. In this paper we do not focus on the transfer time of a single resource, but we consider the whole traffic produced by all the transfers.

In this paper we mainly adopt the model proposed in [22], since it natively support the subdivision of peers among different ASs that will be detailed in Section 4.

3. Modeling Autonomous Systems

We are interested in studying the costs that must be sustained and the gains that can be achieved from the agreements set among the ASs. In particular, starting from a formal description of the network topology, we want to be able to estimate gains and costs for each AS induced by a particular traffic scenario.

3.1. AS topology

We consider a network composed by N ASs. We use a *traffic matrix* $\mathbf{X}$ to represent the data flows between the ASs:

$$\mathbf{X} = \begin{bmatrix} x_{11} & \dots & x_{1N} \\ \vdots & x_{ij} & \vdots \\ x_{N1} & \dots & x_{NN} \end{bmatrix}. \tag{1}$$

Each element x_{ij} represents the amount of traffic exchanged on the logical link between the AS i and the AS j. A logical link between i and j is a BGP session over a physical connection between two gateways. The physical connection might spread over several physical links. The network topology is described by a graph G:

$$G = (V, E_p, E_t, \sigma). \tag{2}$$

Element $V = \{AS_1, \dots, AS_N\}$ contains the vertexes of the graph that corresponds to the N ASs, i.e., the list of AS identifiers. As already mentioned in Section 2 we consider two type of agreements, *pure peering* and *transit*. Two ASs connected with a peering link do not charge any fee each other to communicate over that channel, whereas in a transit agreement the customer node i pays to the seller node j instead. To take this issue into account, edges of the graph are defined as ordered pairs and grouped in two disjoint sets E_p and E_t. For each *peering* agreement between ASs i and j, both the pairs (i, j) and (j, i) are inserted in E_p, while a *transit* agreement is represented from the pair (i, j) in E_t, where i is the AS id of the customer and j the seller's one.

Despite agreements are confidential, it is a reasonable assumption that cost is proportional to the traffic carried on links. These costs are computed using the relations $\sigma : (\mathbb{N} \times \mathbb{N}) \rightarrow \mathbb{R}$. It associates to an ordered pair (i, j) the fee payed from AS_i for a traffic unit carried on the link from AS_i to AS_j. The relation returns this cost if there is a transit agreement between AS_i and AS_j with the first as customer, 0 otherwise.

3.2. Cost and reward matrices

Using the G representation of the AS network we build a *Cost Matrix* $\mathbf{C}^k$ and a *Reward Matrix* $\mathbf{R}^k$ for each AS. A value $c_{ij}^k \neq 0$ means that the AS_k is involved in the communication between i and j, and c_{ij}^k is the cost it pays for a traffic unit that travels from AS_i to AS_j. In the same way the *Reward Matrix* $\mathbf{R}^k$ defines how much AS_k gains when two other ASs communicate.

Algorithm 3.1 describes the function *BuildCostRewardsMatrices()* that computes $\mathbf{C}^k$ and $\mathbf{R}^k$ from the network description G. In Algorithm 3.1 line (i) is used the function *DijkstraModified()* (Algorithm 3.2) to find the shortest path between any pair of ASs. This function is a modified version of the Dijkstra algorithm that computes the shortest path among all sources considering transit and peering agreements to eliminate not allowed routes (see Section 2.2).

The purpose of Algorithm 3.2 is to build the predecessor matrix P. The element P(i,j) of this matrix stores the predecessor of the AS_j on the shortest route to it from AS_i. The function repeats the classical single-source Dijkstra algorithm for all vertices using the *AllowedNeighbors()* (Algorithm 3.2 line (i)) to restrict the neighborhood to the nodes that are on allowed paths. We have used the notation $Dist(i, j)$ to denote the distance to be used in the minimum path algorithm. In our application we have simply set $Dist(i, j) = 1$ if AS_i and AS_j are directly connected. Algorithm 3.3 takes into account that an AS does not announce routes between peering partners, between peering ASs and transit seller or between two seller nodes (Algorithm 3.3 line (ii)). It will announce routes for all its neighbors to its customers nodes (Algorithm 3.3 line (i)) instead.

Once computed all the shortest allowed routes (Algorithm 3.1 line (i)), the matrices $\mathbf{C}^k$ and $\mathbf{R}^k$ are built using the predecessor matrix P to retrieve the path among each pair of ASs. If between two nodes that are on the route from AS_i to AS_j there is a transit agreement, the fees computed using the σ relation are stored in the C_{ij}^k element of the customer AS_k and in the R_{ij}^s reward matrix of the seller AS_s (Algorithm 3.1 lines (ii) and (iii)).

Algorithm 3.1: BUILDCOSTREWARDMATRICES(V, E_p, E_t, σ)

$$
\begin{array}{ll}
P \leftarrow DijkstraModified(V, E_p, E_t) & \text{(i)} \\
\textbf{for each } (i,j) \in (V \times V), i \neq j & \\
\quad \textbf{do} \begin{cases} k \leftarrow j \\ \textbf{while } P(i,k) \neq NULL \\ \quad \textbf{do} \begin{cases} p \leftarrow P(i,k) \\ \textbf{if } (p,k) \in E_t \\ \quad \textbf{do } \left\{ R_{ij}^{k} = \sigma(p,k); \;\; C_{ij}^{p} = \sigma(p,k) \right. \quad \text{(ii)} \\ \textbf{if } (k,p) \in E_t \\ \quad \textbf{do } \left\{ C_{ij}^{k} = \sigma(k,p); \;\; R_{ij}^{p} = \sigma(k,p) \right. \quad \text{(iii)} \\ k \leftarrow p \end{cases} \end{cases} &
\end{array}
$$

Algorithm 3.2: DIJKSTRAMODIFIED(V, E_p, E_t, σ)

$$
\begin{array}{l}
D(i,i) \leftarrow 0, \forall i \in V \\
D(i,j) \leftarrow \infty, \forall (i,j) \in V \times V, i \neq j \\
P(i,j) \leftarrow NULL, \forall (i,j) \in V \times V \\
\textbf{for each } i \in V \\
\quad \textbf{do} \begin{cases} Q \leftarrow V \\ u \leftarrow \arg\min\{D(i,j) : j \in Q\} \\ \textbf{while } (Q \neq \emptyset) \wedge (D(i,u) < \infty) \\ \quad \textbf{do} \begin{cases} Q \leftarrow Q \setminus \{u\} \\ S \leftarrow Q \cap AllowNeighbors(E_p, E_t, P(i,u), u) \quad \text{(i)} \\ D(i,j) \leftarrow \min\{D(i,j), D(i,u) + \\ \qquad Dist(u,j)\}, j \in S \\ P(i,j) \leftarrow \arg\min\{D(i,p) + Dist(p,j)\} : \\ \qquad p \in \{P(i,j), u\}, j \in S \end{cases} \end{cases} \\
\textbf{return } (P)
\end{array}
$$

Algorithm 3.3: ALLOWNEIGHBORS(E_p, E_t, p, u)

$$
\begin{array}{ll}
\textbf{if } (p \equiv NULL) \vee ((p,u) \in E_t) & \\
\quad \textbf{do } neighbors \leftarrow \{j : (u,j) \in (E_p \cup E_t)\} \cup \{j : (j,u) \in (E_p \cup E_t)\} & \text{(i)} \\
\quad \textbf{else} & \\
\quad \textbf{do } neighbors \leftarrow \{j : (j,u) \in E_t)\} & \text{(ii)} \\
\textbf{return } (neighbors) &
\end{array}
$$

For sake of clarity, here we report an example related to the network illustrated in Figure 1. The topology is made of six ASs connected with six links. There are some routes that are not allowed, e.g., $AS_1 \stackrel{L_1}{\rightarrow} AS_2 \stackrel{L_4}{\rightarrow} AS_3$ because AS_2 does not announce routes between its peering partners. Note that the connection between AS_3 and AS_4 is possible since AS_2 gains on the link L_3. As consequence of these forbidden paths, AS_3 results connected only to AS_1, AS_2 and AS_4, but not with AS_5 and AS_6. AS_2 case is particularly interesting in this network because it is directly connected to all

Table 1: Routes from AS_1 to others that involve AS_2

$Route_{12} : AS_1 \overset{L_1}{\to} AS_2$
$Route_{13} : AS_1 \overset{L_2}{\to} AS_4 \overset{L_3}{\to} AS_2 \overset{L_4}{\to} AS_3$
$Route_{15} : AS_1 \overset{L_2}{\to} AS_4 \overset{L_3}{\to} AS_2 \overset{L_5}{\to} AS_5$
$Route_{16} : AS_1 \overset{L_2}{\to} AS_4 \overset{L_3}{\to} AS_2 \overset{L_6}{\to} AS_6$

ASs and involved in all communications that originate from $i \in \{AS_1, AS_2, AS_3\}$ and ends in $j \in \{AS_5, AS_6\}$. Therefore all routes that involve AS_2 and a transit agreement influence its cost and reward matrices. In Table 1 we enumerate all the routes from AS_1 to others that involve AS_2. When AS_1 sends a packet to AS_6, it follows the $Route_{16}$, so AS_2 gains a quantity $\sigma(4, 2)$ from AS_4 for the usage of the link L_3 but pays $\sigma(2, 6)$ to AS_6 for the traffic that travels on L_6. In Figure 1 we report the C^2 and R^2 matrices computed for the AS_2.

Once computed the C^k and R^k matrices for each AS, it is simple to retrieve the total cost C_k sustained and the total reward R_k for AS_k simply using:

$$C_k = \vec{1} \cdot (X \circ C^k) \cdot \vec{1}' \cdot \text{ and } R_k = \vec{1} \cdot (X \circ R^k) \cdot \vec{1}' \cdot$$

where $\circ$ defines the entry wise product between matrices and $\cdot$ is the standard row-by-column matrix product.

4. P2P resource diffusion

In this section we describe the probabilistic model of the P2P traffic traveling among the considered ASs. We first define the peer-to-peer scenario (Section 4.1), then we provide the analytical representation of peers (Section 4.2). Finally, we study the evolution of the system in order to characterize the resource diffusion (Section 4.3).

4.1. Network scenario

We consider that peers are distributed across N different ASs, and we assume that the total number of peers in the system is a discrete random variable that follows a given probability distribution $p(m)$. We express this distribution with its generating function $\mathcal{G}(z)$:

$$\mathcal{G}(z) = \sum_{m=0}^{\infty} p(m) z^m \tag{3}$$

We only take into account peers that can participate in the diffusion, i.e., peers that either hold or request the resource. We denote by s_i the probability that a peer is in the i-th AS. In each AS peers are divided into three different classes: a) peers holding the resource and available for sharing it, b) peers requiring the resource, and c) peers holding the resource, but not sharing it, i.e., freeloaders [8]. The class of each peer is

Table 2: Model Notations

Notation	Description
N	Number of Autonomous Systems (AS)
s_i	Probability that a peer belongs to the i-th AS
$\mathcal{G}(z)$	Generating function of the distribution of the number of peers in the network
α_i	Probability that a peer in the i-th AS holds the resource
β_i	Probability that a peer in the i-th AS wants the resource
γ_i	Probability that a peer in the i-th AS holds the resource but does not share it
ξ_i	Probability that a peer in the i-th AS does not share the resource after getting it
n_i	Number of peers holding the resource in the i-th AS
p_i	Number of peers requiring the resource in the i-th AS
q_i	Number of peers holding the resource but not sharing it in the i-th AS
u_i	Generating function of the number of peers holding the resource in the i-th AS
v_i	Generating function of the number of peers requiring the resource in the i-th AS
w_i	Generating function of the number of peers holding but not sharing it in the i-th AS
σ_{ij}	Cost (Fee) payed by AS_i for a traffic unit carried on the link from AS_i to AS_j
θ_i	Resource splitting factor between owned and searched in the i-th AS
C^k	Cost matrix for AS_k
c_{ij}^k	The cost for AS_k when AS_i and AS_j communicate
R^k	Reward matrix for AS_k
r_{ij}^k	The reward for AS_k when AS_i and AS_j communicate
wgt_{ij}	The weight assigned by AS_i to AS_j for the Weighted Policy

determined randomly, according to a given initial probability: α_i for class a), β_i for class b) and γ_i for class c) (with $\alpha_i + \beta_i + \gamma_i = 1$). Not that γ_i has no impact on the system behavior, but it is of interest since it allows us to investigate of the number of freeloaders. The number of peers of the three classes are respectively denoted by n_i, p_i and q_i. We denote by ξ_i the probability that a peer that gets the resource decides to not share it. Peers requiring the resource can either get it from peers lying in the the same AS or from peers belonging to others ASs. All model notations are summarized in Table 2.

4.2. The model

We represent the P2P system by introducing the distribution of the number of peers and its corresponding generating function.

We call $\Pi(n_1...n_N, p_1...p_N, q_1...q_N)$ the joint distribution of the number of peers in each class, for each of the N ASs. The generating function $g(\cdot)$ of this distribution can be computed from parameters $\mathcal{G}(z)$, s_i, α_i, β_i and γ_i as:

$$g(u_1...u_N, v_1...v_N, w_1...w_N) = \mathcal{G}\Big(\sum_{i=1}^{N} s_i\Big[\alpha_i u_i + \beta_i v_i + \gamma_i w_i\Big]\Big). \tag{4}$$

An intuitive interpretation of Equation (4), is that each peer randomly chooses both its AS and its class with probability $s_i\alpha_i$, $s_i\beta_i$ or $s_i\gamma_i$, which corresponds to $z = \sum_{i=1}^{N} s_i\Big[\alpha_i u_i + \beta_i v_i + \gamma_i w_i\Big]$.

The marginal distribution corresponding to the i-th AS is defined as:

$$\Pi_i(n_i, p_i, q_i) = \sum_{\substack{n_1\cdots n_{i-1},\, n_{i+1}\cdots n_N,\\ p_1\cdots p_{i-1},\, p_{i+1}\cdots p_N,\\ q_1\cdots q_{i-1},\, q_{i+1}\cdots q_N}} \Pi(n_1...n_N, p_1...p_N, q_1...q_N) \tag{5}$$

We call $g_i(u_i, v_i, w_i)$ the generating function of $\Pi_i(n_i, p_i, q_i)$. Using the properties of the generating function and (4), we have that:

$$\begin{aligned} g_i(u_i, v_i, w_i) = g(1...1, u_i, 1...1, 1...1, v_i, 1...1, 1...1, w_i, 1...1) = \\ = \mathcal{G}\Big(s_i\Big[\alpha_i u_i + \beta_i v_i + \gamma_i w_i\Big] + 1 - s_i\Big) \end{aligned} \tag{6}$$

where we set to 1 all the transformed variables except the ones corresponding to the i-th AS.

We can compute the probability that a peer in the i-th AS holds the resource, given that the AS is not empty (by empty we mean that there are no peers that can participate in the resource diffusion as mentioned in Section 4.1) as:

$$\bar{\alpha_i} = \sum_{n_i+p_i+q_i \neq 0} \frac{n_i}{n_i + p_i + q_i} \Pi_i(n_i, p_i, q_i). \tag{7}$$

It can be shown that $\bar{\alpha_i}$ can be computed in the following way:

$$\bar{\alpha_i} = \int_0^1 \Big[\frac{\partial g_i(u_i, v_i, w_i)}{\partial u_i}\Big]_{u_i=y, v_i=y, w_i=y} dy. \tag{8}$$

$\Diamond$**Proof:** We can show that we can obtain (7) from (8) by solving step by step the right term of the equation. Starting from explicit form of 5 we have:

$$g_i(u_i, v_i, w_i) = \sum_{n_i, p_i, q_i} u_i^{n_i} v_i^{p_i} w_i^{q_i} \Pi_i(n_i, p_i, q_i)$$

$$\frac{\partial g_i(u_i, v_i, w_i)}{\partial u_i} = \sum_{n_i, p_i, q_i} n_i u_i^{n_i - 1} v_i^{p_i} w_i^{q_i} \Pi_i(n_i, p_i, q_i)$$

$$\begin{aligned} \Big[\frac{\partial g_i(u_i, v_i, w_i)}{\partial u_i}\Big]_{u_i=y, v_i=y, w_i=y} = \\ \sum_{n_i, p_i, q_i} n_i y^{n_i + p_i + q_i - 1} \Pi_i(n_i, p_i, q_i) \end{aligned}$$

$$\begin{aligned} \int \Big[\frac{\partial g_i(u_i, v_i, w_i)}{\partial u_i}\Big]_{u_i=y, v_i=y, w_i=y} dy = \\ \sum_{n_i, p_i, q_i} \frac{n_i}{n_i + p_i + q_i} y^{n_i + p_i + q_i} \Pi_i(n_i, p_i, q_i) + c \end{aligned}$$

Equation (8) is proven by computing the integral between 0 and 1.$\diamondsuit$

If we calculate (8) with the definition (6) we get:

$$\bar{\alpha_i} = \alpha_i(1 - g_i(0,0,0)) \tag{9}$$

Hence we can write:

$$\alpha_i = \frac{\bar{\alpha_i}}{(1 - g_i(0,0,0))} = \frac{\bar{\alpha_i}}{(1 - \mathcal{G}(1 - s_i))} \tag{10}$$

The term $1 - \mathcal{G}(1 - s_i)$ corresponds to the probability that the i-th AS is not empty. The same computation can be made for β and γ:

$$\beta_i = \frac{\bar{\beta}_i}{(1 - g_i(0,0,0))},\ \bar{\beta}_i = \int_0^1 \left[\frac{\partial g_i(u_i, v_i, w_i)}{\partial v_i}\right]_{\substack{u_i = y \\ v_i = y \\ w_i = y}} dy \tag{11}$$

$$\gamma_i = \frac{\bar{\gamma_i}}{(1 - g_i(0,0,0))},\ \bar{\gamma_i} = \int_0^1 \left[\frac{\partial g_i(u_i, v_i, w_i)}{\partial w_i}\right]_{\substack{u_i = y \\ v_i = y \\ w_i = y}} dy \tag{12}$$

4.3. System dynamics

We now study the evolution of parameters α_i, β_i and γ_i, and show how they characterize the resource diffusion in the system. In particular, we model the resource diffusion among the ASs with an *embedded time* process: time is not considered explicitly, instead is modeled by a discrete variable m that increases of one unit whenever a resource transfer is completed. With this assumption we compute α_i^m, β_i^m and γ_i^m, that correspond to the values of parameters α_i, β_i and γ_i at time m.

Parameters α and β vary only due to a resource transfer. For sake of simplicity, in this paper we neglect peers that give up requesting the resource and peers that quit sharing it. However these assumptions could be easily removed by adding new parameters and different equations on α and β. We expect that as time tends to the infinity, every requests will be satisfied, that is (for any i-th AS):

$$\lim_{m\to\infty} \alpha_i^m = \alpha_i^0 + (1 - \xi_i)\beta_i^0 \tag{13}$$

$$\lim_{m\to\infty} \beta_i^m = 0 \tag{14}$$

$$\lim_{m\to\infty} \gamma_i^m = \gamma_i^0 + \xi_i\beta_i^0 \tag{15}$$

where α_i^0, β_i^0 and γ_i^0 represent the initial system parameters.We are interested in studying the evolution of α_i^m, β_i^m and γ_i^m until all transfers are completed. Note that changes in these parameters affect the joint distribution of the number of peers per class, i.e. $\Pi^m(n_1..n_N, p_1..p_N, q_1..q_N)$.

We can define $\bar{\alpha}_i^{m+1}$ as function of the system parameters at time m:

$$
\begin{aligned}
\bar{\alpha}_i^{m+1} = & \sum_{\substack{(\sum_k p_k = 0 \vee \\ \sum_k n_k = 0) \\ n_i + p_i + q_i \neq 0}} \frac{n_i}{n_i + p_i + q_i} \Pi^m(n_1..n_N, p_1..p_N, q_1..q_N) + \\
& + \sum_{\substack{(\sum_k p_k \neq 0 \wedge \\ \sum_k n_k \neq 0) \\ n_i + p_i + q_i \neq 0}} \left[\frac{n_i + 1}{n_i + p_i + q_i} \frac{p_i}{\sum_k p_k}(1 - \xi_i) + \right. \\
& \left. + \frac{n_i}{n_i + p_i + q_i} \left(1 - \frac{p_i}{\sum_k p_k}(1 - \xi_i) \right) \right] \cdot \\
& \cdot \; \Pi^m(n_1..n_N, p_1..p_N, q_1..q_N)
\end{aligned} \tag{16}
$$

The first addendum on the r.h.s. accounts for the case in which no transfer occurs since either all requests in the system have been satisfied and there are no more peers requiring the resource ($\sum_k p_k = 0$), or there are no resources in the system ($\sum_k n_k = 0$). The second addendum on the r.h.s. considers the case where a resource is actually transferred. If the destination of the transfer is the i-th AS and the considered peer is not a freeloader (with probability $\frac{p_i}{\sum_k p_k}(1-\xi_i)$), then n_i is increased by one (first term in square brackets), otherwise n_i remains constant (second term in square brackets). By developing (16) we obtain:

$$
\begin{aligned}
\bar{\alpha}_i^{m+1} = & \sum_{n_i + p_i + q_i \neq 0} \frac{n_i}{n_i + p_i + q_i} \Pi^m(n_1..n_N, p_1..p_N, q_1..q_N) + \\
& + \sum_{\substack{(\sum_k p_k \neq 0 \wedge \\ \sum_k n_k \neq 0) \\ n_i + p_i + q_i \neq 0}} \frac{1}{n_i + p_i + q_i} \frac{p_i}{\sum_k p_k} \cdot \\
& . \; \Pi^m(n_1..n_N, p_1..p_N, q_1..q_N)(1 - \xi_i)
\end{aligned} \tag{17}
$$

From (7) we can write:

$$
\bar{\alpha}_i^{m+1} = \bar{\alpha}_i^m + \Delta_i^m (1 - \xi_i) \tag{18}
$$

where we define

$$
\begin{aligned}
\Delta_i^m = & \sum_{\substack{(\sum_k p_k \neq 0 \wedge \\ \sum_k n_k \neq 0) \\ n_i + p_i + q_i \neq 0}} \frac{1}{n_i + p_i + q_i} \frac{p_i}{\sum_k p_k} \Pi^m(n_1..n_N, p_1..p_N, q_1..q_N) \\
= & \sum_{\substack{\sum_k p_k \neq 0 \\ n_i + p_i + q_i \neq 0}} \frac{1}{n_i + p_i + q_i} \frac{p_i}{\sum_k p_k} \Pi^m(n_1..n_N, p_1..p_N, q_1..q_N) \\
- & \sum_{\substack{\sum_k p_k \neq 0 \\ n_i + p_i + q_i \neq 0}} \frac{1}{p_i + q_i} \frac{p_i}{\sum_k p_k} \Pi^m(0..0, p_1..p_N, q_1..q_N).
\end{aligned} \tag{19}
$$

$\Delta_i^m(1-\xi_i)$ represents the variation of $\bar{\alpha}_i$ at time m. In the same way the evolution of $\bar{\beta}_i$ and $\bar{\gamma}_i$ can be calculated as:

$$\bar{\beta}_i^{m+1} = \bar{\beta}_i^m - \Delta_i^m \tag{20}$$

$$\bar{\gamma}_i^{m+1} = \bar{\gamma}_i^m + \Delta_i^m \xi_i \tag{21}$$

Δ_i^m can be computed exploiting the generating function representation of the number of peers in the system. We define $\bar{p}_i = \sum_{k \neq i} p_k$ and we denote with $\bar{v}_i$ its corresponding transformed variable. It can be shown that:

$$\Delta_i^m = \int_0^1 \left[\int_0^1 \left[\frac{\partial}{\partial v_i} \hat{g}_i(u_i, v_i, w_i, \bar{v}_i) \right]_{\substack{v_i = xy \\ \bar{v}_i = x}} dx \right]_{\substack{u_i = y \\ w_i = y}} dy. \tag{22}$$

where

$$\begin{aligned} \hat{g}_i(u_i, v_i, w_i, \bar{v}_i) &= g(1...1, u_i, 1...1, \bar{v}_i...\bar{v}_i, v_i, \bar{v}_i...\bar{v}_i, 1...1, w_i, 1...1) - \\ & \quad g(0...0, \bar{v}_i...\bar{v}_i, v_i, \bar{v}_i...\bar{v}_i, 1...1, w_i, 1...1) = \\ &= \mathcal{G}\Big(1 + s_i\Big[\alpha_i(u_i - 1) + \beta_i(v_i - \bar{v}_i) + \gamma_i(w_i - 1)\Big] + B(\bar{v}_i - 1)\Big) - \\ & \quad \mathcal{G}\Big(1 - A + s_i\Big[\beta_i(v_i - \bar{v}_i) + \gamma_i(w_i - 1)\Big] + B(\bar{v}_i - 1)\Big) \end{aligned} \tag{23}$$

and $A = \sum_k s_k \alpha_k$ and $B = \sum_k s_k \beta_k$.

$\diamondsuit$**Proof:** By definition of generating function we have:

$$\hat{g}_i(u_i, v_i, w_i, \bar{v}_i) = \sum_{n_i, p_i, q_i, \bar{p}_i} u_i^{n_i} v_i^{p_i} w_i^{q_i} \bar{v}_i^{\bar{p}_i} \hat{\Pi}_i(n_i, p_i, q_i, \bar{p}_i) \tag{24}$$

Elaborating (23) we obtain:

$$\begin{aligned} & g(1...1, u_i, 1...1, \bar{v}_i...\bar{v}_i, v_i, \bar{v}_i...\bar{v}_i, 1...1, w_i, 1...1) - \\ & \quad g(0...0, \bar{v}_i...\bar{v}_i, v_i, \bar{v}_i...\bar{v}_i, 1...1, w_i, 1...1) = \\ & \sum_{n_i, p_i, q_i, \bar{p}_i} \sum_{\substack{n_1 \cdots n_{i-1},\, n_{i+1} \cdots n_N, \\ p_1 \cdots p_{i-1},\, p_{i+1} \cdots p_N \wedge \\ \sum_k P_k = \bar{p}_i, \\ q_1 \cdots q_{i-1},\, q_{i+1} \cdots q_N}} \Pi^m(n_1..n_N, p_1..p_N, q_1..q_N) u_i^{n_i} v_i^{p_i} w_i^{q_i} \bar{v}_i^{\bar{p}_i} - \\ & \sum_{p_i, q_i, \bar{p}_i} \sum_{\substack{p_1 \cdots p_{i-1},\, p_{i+1} \cdots p_N \wedge \\ \sum_k P_k = \bar{p}_i, \\ q_1 \cdots q_{i-1},\, q_{i+1} \cdots q_N}} \Pi^m(0..0, p_1..p_N, q_1..q_N) v_i^{p_i} w_i^{q_i} \bar{v}_i^{\bar{p}_i} \end{aligned} \tag{25}$$

Comparing (25) with (24) we have:

$$\begin{aligned} \hat{\Pi}_i = & \sum_{\substack{n_1 \cdots n_{i-1},\, n_{i+1} \cdots n_N, \\ p_1 \cdots p_{i-1},\, p_{i+1} \cdots p_N \wedge \sum_k P_k = \bar{p}_i, \\ q_1 \cdots q_{i-1},\, q_{i+1} \cdots q_N}} \Pi^m(n_1..n_N, p_1..p_N, q_1..q_N) - \\ & \sum_{\substack{p_1 \cdots p_{i-1},\, p_{i+1} \cdots p_N \wedge \sum_k P_k = \bar{p}_i, \\ q_1 \cdots q_{i-1},\, q_{i+1} \cdots q_N}} \Pi^m(0..0, p_1..p_N, q_1..q_N) \end{aligned} \tag{26}$$

By using a technique similar to the one used in the proof of (8) we can compute from (22):

$$\frac{\partial \hat{g}_i}{\partial v_i} = \sum_{n_i, p_i, q_i, \bar{p}_i} p_i u_i^{n_i} v_i^{p_i - 1} w_i^{q_i} \bar{v}_i^{\bar{p}_i} \hat{\Pi}_i(n_i, p_i, q_i, \bar{p}_i)$$

$$\left[\frac{\partial \hat{g}_i(u_i, v_i, w_i, \bar{v}_i)}{\partial v_i}\right]_{\substack{v_i = xy \\ \bar{v}_i = x}} =$$

$$\sum_{n_i,p_i,q_i,\bar{p}_i} p_i u_i^{n_i} (xy)^{p_i - 1} w_i^{q_i} x^{\bar{p}_i} \hat{\Pi}_i(n_i, p_i, q_i, \bar{p}_i) =$$

$$\sum_{n_i,p_i,q_i,\bar{p}_i} p_i u_i^{n_i} x^{p_i + \bar{p}_i - 1} w_i^{q_i} y^{p_i - 1} \hat{\Pi}_i(n_i, p_i, q_i, \bar{p}_i)$$

$$\int \left[\frac{\partial \hat{g}_i(u_i, v_i, w_i, \bar{v}_i)}{\partial v_i}\right]_{\substack{v_i = xy \\ \bar{v}_i = x}} dx =$$

$$\sum_{n_i,p_i,q_i,\bar{p}_i} \frac{p_i}{p_i + \bar{p}_i} x^{p_i + \bar{p}_i} u_i^{n_i} w_i^{q_i} y^{p_i - 1} \hat{\Pi}_i(n_i, p_i, q_i, \bar{p}_i) + c$$

$$\int_0^1 \left[\frac{\partial \hat{g}_i(u_i, v_i, w_i, \bar{v}_i)}{\partial v_i}\right]_{\substack{v_i = xy \\ \bar{v}_i = x}} dx =$$

$$\sum_{n_i,q_i,p_i+\bar{p}_i \neq 0} \frac{p_i}{p_i + \bar{p}_i} u_i^{n_i} w_i^{q_i} y^{p_i - 1} \hat{\Pi}_i(n_i, p_i, q_i, \bar{p}_i)$$

$$\left[\int_0^1 \left[\frac{\partial \hat{g}_i(u_i, v_i, w_i, \bar{v}_i)}{\partial v_i}\right]_{\substack{v_i = xy \\ \bar{v}_i = x}} dx\right]_{\substack{u_i = y \\ w_i = y}} =$$

$$\sum_{n_i,q_i,p_i+\bar{p}_i \neq 0} \frac{p_i}{p_i + \bar{p}_i} y^{n_i + p_i + q_i - 1} \hat{\Pi}_i(n_i, p_i, q_i, \bar{p}_i)$$

$$\int \left[\int_0^1 \left[\frac{\partial \hat{g}_i(u_i, v_i, w_i, \bar{v}_i)}{\partial v_i}\right]_{\substack{v_i = xy \\ \bar{v}_i = x}} dx\right]_{\substack{u_i = y \\ w_i = y}} dy =$$

$$\sum_{n_i,q_i,p_i+\bar{p}_i \neq 0} \frac{p_i}{p_i + \bar{p}_i} \frac{1}{n_i + p_i + q_i} y^{n_i + p_i + q_i} \hat{\Pi}_i(n_i, p_i, q_i, \bar{p}_i) + c$$

$$\int_0^1 \left[\int_0^1 \left[\frac{\partial \hat{g}_i(u_i, v_i, w_i, \bar{v}_i)}{\partial v_i}\right]_{\substack{v_i = xy \\ \bar{v}_i = x}} dx\right]_{\substack{u_i = y \\ w_i = y}} dy = \tag{27}$$

$$\sum_{n_i+p_i+q_i \neq 0, p_i+\bar{p}_i \neq 0} \frac{p_i}{p_i + \bar{p}_i} \frac{1}{n_i + p_i + q_i} \hat{\Pi}_i(n_i, p_i, q_i, \bar{p}_i)$$

If we insert (26) into (27) we obtain (22).$\diamondsuit$

By calculating (22) with (23) we obtain:

$$\begin{aligned}\Delta_i^m = \int_0^1 \frac{s_i\beta_i}{s_i\beta_i(y-1) + B} \Big[& \mathcal{G}\left(1 + s_i(y-1)\right) - \\ & \mathcal{G}\left(1 - B + s_i(1-\beta_i)(y-1)\right) - \\ & \mathcal{G}\left(1 - A + s_i(1-\alpha_i)(y-1)\right) + \\ & \mathcal{G}\left(1 - A - B + s_i\gamma_i(y-1)\right) \Big] dy.\end{aligned} \tag{28}$$

Finally, from (18), (20), (21) and (28) we are able to compute $\alpha_i^m, \beta_i^m, \gamma_i^m$ for any step m and for each AS i. Given the initial parameters α_i^0, β_i^0 and γ_i^0 we have, by applying

(10), (11) and (12), that:

$$\alpha_i^m = \frac{\bar{\alpha}_i^m}{(1 - \mathcal{G}(1 - s_i))} \quad (29)$$

$$\beta_i^m = \frac{\bar{\beta}_i^m}{(1 - \mathcal{G}(1 - s_i))} \quad (30)$$

$$\gamma_i^m = 1 - (\bar{\alpha}_i^m + \bar{\beta}_i^m) \quad (31)$$

5. Traffic among Autonomous Systems

The goal is to compute the traffic related to the resource across the N ASs by considering three different search policies: uniform, internal first and weighted search.

5.1. Uniform search

We define X_{ji} as the probability that there is a resource transfer from the AS j to the AS i as reported in Table 3, row A. Note that conditions that allows a resource transfer from AS j to i are $n_j \neq 0$ and $p_i \neq 0$. Since n_j and p_i are both to the numerator, it is equivalent to use $\sum_{k=1}^{N} n_k \neq 0$ and $\sum_{k=1}^{N} p_k \neq 0$.
Using techniques similar to the proof of Equation (8) we can obtain the results shown in Table 4, row A.
By calculating Equation reported in Table 4 row A with Equation (4) we obtain the expression indicated in row A of Table 5. Note that $\sum_{i,j} X_{ji}$ is equal to the probability that there is at least one peer holding the resource and one peer requiring it in the whole system.

5.2. Internal search first

Let suppose the AS searches resources first among its peers and if it is not present it will search in the others ASs. Then the definition of X can be split in the following two cases as illustrated in Table 3, row $B1$ to consider the internal search first and row $B2$ to describe the external search. By using the generating function we can write these equations as shown in Table 4 rows $B1$ and $B2$.
The derivations of the previous equations can be easily obtained using a technique similar to the one used to prove Equation (8), we have omitted for brevity.
Finally, by calculating equations indicated in Table 4 rows $B1$ and $B2$ with Equation (4) we obtain the expression shown in row B of Table 5.

5.3. Weighted search

Each peer in AS i seeks the needed resources with higher probability in the AS k with higher weight wgt_{ik}. The weight assigned by an AS to itself and to others can be set according to different criteria. For instance, an AS could prefer shorter paths to longer ones when communicating with other ASs. These expression are reported in rows C of Tables 3, 4 and 5.

Table 3: Different policies: initial expressions

A	$X_{ji} = \sum_{\substack{n_1...n_N,\, p_1...p_N,\, q_1...q_N : \\ \sum_{k=1}^N n_k \neq 0,\, \sum_{k=1}^N p_k \neq 0}} \frac{n_j}{\sum_{k=1}^N n_k} \frac{p_i}{\sum_{k=1}^N p_k} \Pi(n_1...n_N, p_1...p_N, q_1...q_N).$
B1	$X_{ii} = \sum_{\substack{n_1...n_N,\, p_1...p_N,\, q_1...q_N : \\ n_i \neq 0,\, \sum_{k=1}^N p_k \neq 0}} \frac{p_i}{\sum_{k=1}^N p_k} \Pi(n_1...n_N, p_1...p_N, q_1...q_N).$
B2	$X_{ji} = \sum_{\substack{n_1...n_N,\, p_1...p_N,\, q_1...q_N : \\ n_i = 0, \\ \sum_{k=1}^N p_k \neq 0,\, \sum_{h=1}^N n_h \neq 0}} \frac{n_j}{\displaystyle\sum_{k=1,k\neq i}^N n_k} \frac{p_i}{\displaystyle\sum_{k=1}^N p_k} \Pi(n_1...n_N, p_1...p_N, q_1...q_N), i \neq j.$
C	$X_{ji} = \sum_{\substack{n_1...n_N,\, p_1...p_N,\, q_1...q_N : \\ \sum_{k=1}^N n_k \neq 0,\, \sum_{k=1}^N p_k \neq 0}} \frac{wgt_{ij} n_j}{\sum_{k=1}^N wgt_{ik} n_k} \frac{p_i}{\sum_{k=1}^N p_k} \Pi(n_1...n_N, p_1...p_N, q_1...q_N).$

Table 4: Different policies: generating functions

A	$X_{ji} = \int_0^1 \int_0^1 \left[\frac{\partial}{\partial u_j} \frac{\partial}{\partial v_i} g(u_1...u_N, v_1...v_N, w_1...w_N) \right]_{\substack{u_1...u_n = x \\ v_1...v_n = y \\ w_1...w_N = 1}} dx\, dy.$
B1	$X_{ii} = \int_0^1 \Big[\frac{\partial}{\partial v_i} \Big(g(u_1...u_N, v_1...v_N, w_1...w_N) + -g(u_1..u_{j-1}, 0, u_{j+1}..u_N, v_1...v_N, w_1...w_N) \Big) \Big]_{\substack{u_1...u_n = x \\ v_1...v_n = y \\ w_1...w_N = 1}} dx\, dy.$
B2	$X_{ji} = \int_0^1 \int_0^1 \left[\frac{\partial}{\partial u_j} \frac{\partial}{\partial v_i} g(u_1..u_{j-1}, 0, u_{j+1}..u_N, v_1...v_N, w_1...w_N) \right]_{\substack{u_1...u_n = x \\ v_1...v_n = y \\ w_1...w_N = 1}} dx\, dy.$
C	$X_{ji} = \int_0^1 \int_0^1 \left[\frac{\partial}{\partial \hat{u}_j} \frac{\partial}{\partial v_i} \left[g(u_1...u_N, v_1...v_N, w_1...w_N) \right]_{u_k = \hat{u}^{wgt_{ik}}} \right]_{\substack{u_1...u_N = x \\ v_1...v_N = y \\ w_1...w_N = 1}} dx\, dy.$

Table 5: Different policies: final expressions

A	$X_{ji} = \frac{\alpha_j s_j}{A} \frac{\beta_i s_i}{B} \left[1 - \mathcal{G}(1-A) - \mathcal{G}(1-B) + \mathcal{G}(1-A-B) \right]$
B	$X_{ji} = \begin{cases} \frac{\alpha_i s_i}{A - \alpha_j s_j} \frac{\beta_j s_j}{B} \left[\mathcal{G}(1-\alpha_j s_j) - \mathcal{G}(1-A) - \mathcal{G}(1-B-\alpha_j s_j) + \mathcal{G}(1-A-B) \right] & j \neq i \\ \frac{\beta_j s_j}{B} \left[1 - \mathcal{G}(1-\alpha_j s_j) - \mathcal{G}(1-B) - \mathcal{G}(1-B-\alpha_j s_j) \right] & j = i \end{cases}$
C	$X_{ji} = \frac{\beta_i s_i}{B} \int_0^1 \alpha_j s_j wgt_{ij} x^{wgt_{ij}-1} \left[\mathcal{G}'(1 - A + \sum_{k=1}^N \alpha_k s_k x^{wgt_{ik}}) - \mathcal{G}'(1 - A - B + \sum_{k=1}^N \alpha_k s_k x^{wgt_{ik}}) \right] dx$

6. Considering more resources

To create a more realistic scenario we suppose that each peer holds more than one resource type. We denote with N_{max} the maximum number of resource types available in the whole system. Each one is characterized by a popularity level that describes the probability that a peer has such a resource. As stated in [12] the popularity of the resources follows a Zipf Mandelbrot [20] distribution. The resource distribution is characterized by three parameters: q_Z, s_Z and $\bar{n}_Z(i)$. The first two parameters define the Zipf Mandelbrot distribution and they are constant for the whole system, whereas $\bar{n}_Z(i)$ represents the mean number of resources per peer, and we allow it to depend on the autonomous system i. In particular, let us define $f_i(k)$ with $1 \leq k \leq N_{Max}$ as the probability that a peer in the $i-th$ AS holds resource k, computed as:

$$f_i(k) = \frac{\bar{n}_Z(i) \cdot H}{(k + q_Z)^{s_Z}} \tag{32}$$

with

$$H = \frac{1}{\sum_{l=1}^{N_{Max}} \frac{1}{(l + q_Z)^{s_Z}}}$$

It is easy to show that with (32) and the above definition we have $\sum_{l=1}^{N_{Max}} f_i(l) = \bar{n}_Z(i)$. Note that since $f_i(k)$ is a probability distribution the following constraints must hold:

$$\bar{n}_Z(i) \leq \frac{(1 + q_Z)^{s_Z}}{H}.$$

In a scenario with $N_{Max} > 1$ resources we compute equations (29), (30) and (31) for any k-th resource type. We introduce the notation $\alpha_i^m(k)$ to study the evolution of the $k-th$ resources in the $i-th$ AS at step m. The function reported in (32) is used to determine the initial values of $\alpha_i^m(k)$, $\beta_i^m(k)$ and $\gamma_i^m(k)$ by equations (33), (34), and (35) respectively:

$$\alpha_i^0(k) = f_i(k)\theta_i \tag{33}$$

$$\beta_i^0(k) = f_i(k)(1 - \theta_i) \tag{34}$$

$$\gamma_i^0(k) = 1 - f_i(k) \tag{35}$$

where θ_i is the factor that splits the resource from owned (θ) to searched ($1-\theta$) for the i-th AS. The evolution of α, β and γ is successively computed from (18), (20) and (21) for any i-th AS and k-th resource.

7. Model validation

The whole model that accounts for system dynamics, searching policies that drive traffic crossing the ASs, and more than one resource type, is validated using PeerSim [14]. It is a discrete event simulator providing a collection of features that help the implementation and analysis of network protocols or multi-agent simulations. Here we present a comparison between the model and the simulation for the weighted case (Section 5.3) with the P2P overlay distributed over three ASs as shown in Figure 4. Note that, the uniform strategy can be considered a special case where all weights are equal.

In the simulated scenario the whole peer population is grouped into ASs of different sizes. The number of resource types in the system is fixed to $N_{Max} = 10$. Each peer can be interested in a resource according to the resource popularity. Resource's popularity is modeled according to the Zipf-Mandelbrot [20] discrete distribution as explained in Section 6.

Each peer of the system has two lists: one for owned resources and one for resources in which is interested. The elements of both lists are assigned randomly during the initialization by using Algorithm 7.1.

Algorithm 7.1: RESOURCEASSIGNEMENTS($AS, Resource$)

- **for each** $AS_i \in AS$
- **do**
  - **for each** $peer_j \in AS_i$
  - **do**
    - $owned \leftarrow \emptyset$
    - $interest \leftarrow \emptyset$
    - **for each** $res_r \in Resource$
    - **do**
      - **if** $(rnd < ZIPF(res_r, AS_i))$
      - **do**
        - **if** $(rnd < \theta_i)$
        - **do** $owned \leftarrow owned \cup res_r$
        - **else**
        - **do** $interest \leftarrow interest \cup res_r$

The function rnd generates a random number uniformly distributed between 0 and 1 and $ZIPF$ is a function that computes equation (32).

The duration of simulation is subdivided into rounds, and each round into time slots. A round has exactly a number of time slots equal to the number of peers active in the overlay. In this way all peers are scheduled exactly once for one simulation round. When scheduled, a peer makes a request to another for a specific resource contained in its $interest$ list.

The search is made according to the chosen policy. In our simulation each peer can use both the uniform and the weighted strategy explained in Section 5. When a peer j belonging to an AS_i finds a resource r in a peer k in AS_l then the number of transfers from AS_l to AS_i is increased. At the end of the round the resource r is removed from the $interest$ list of peer j and it is added to $owned$ list with probability $1 - \xi_i$ (see Section 4.1). The peer hides the resource (i.e., it is not added to $owned$ list) with probability ξ_i to model the freeloader behavior.

Table 6: Simulation parameters

Description	Value
n_{tot}	5000
n_1, n_2, n_3	2500, 1800, 700
N_{Max}	10
s_Z, q_Z	1, 1
$\theta_1, \theta_2, \theta_3$	0.5, 0.2, 0.7
$n_Z(1), n_Z(2), n_Z(3)$	2.5, 2.5, 2.5

7.1. Results and Parameters Sensitivity

The simulation parameters are indicated in Table 6. The network is composed of 3 ASs with a total number of peers (n_{tot}) equal to 5000 distributed as follow: 2500 in AS_1, 1800 in AS_2, and 700 in AS_3. The number of resource types (N_{Max}) spread in the system is equal to 10. s_Z and q_Z are respectively the skewness factor and the plateau factor of the Zipf-Mandelbrot distribution. For any resource type we have that AS_1 has the same number of holders and searchers ($\theta_1 = 0.5$), AS_2 has a higher number of searchers ($\theta_1 = 0.2$), whereas AS_3 has a higher number of holders ($\theta_1 = 0.7$). All the ASs share the same n_Z value, that is 2.5, that is the mean number of resources per peer.

We assign the following weight matrix WGT:

$$WGT = \begin{bmatrix} 4 & 2 & 1 \\ 1 & 2 & 1 \\ 1 & 2 & 4 \end{bmatrix}$$

where element wgt_{ij} corresponds to the weight that the AS_i assign to AS_j. According to this setting of WGT, each peer seeks the needed resources with higher probability in its same AS, and prefers shorter paths to longer ones when communicating with other ASs.

In Figures 5, 6 and 7 we show the comparison between the evolution over time of model and simulator. The simulator outcomes are the mean of 100 runs and the 95% confidence interval is reported. As we can see the model outcomes are always included in the 95% confidence interval of simulation results. Further confirmation of previous results comes from Figure 8 where we compared the final steady state of simulation and model.

Figure 5 reports the data related to transfers from AS_1: internal traffic is the highest since $wgt_{11} = 4$ and the AS_1 has a large number of peers sharing the resources ($n_1 = 2500$ and $\theta_1 = 0.5$. The traffic due to AS_3 requests is low for two reasons: most of AS_3 peers hold the resources ($\theta_3 = 0.7$), and $wgt_{33} > wgt_{32} > wgt_{31} >$ making most of AS_3 requests seek first internal resources and then the ones of AS_2. Also in AS_2 most of the traffic from AS_2 (Figure 6) is internal whereas the traffic directed to the others is marginal, since AS1 and AS3 have a larger number of resources. The traffic coming from AS_3 (Figure 7) is mostly addressed to AS_2 that requires many resources ($\theta_2 = 0.2$), and is less routed forward AS_1 that assigns to AS_3 the lowest weight ($wgt_{13} = 1$).

8. Experiments

In this section, we conduct several experiments using both the P2P resource diffusion model (Sections 4, 5, and 6) and the AS-level network model (Section 3). In particular, the aim of this section is to present how the proposed results can be used to study the costs that different ASs can achieve using different resource location protocols and different parameter settings in the P2P application. We compute costs and rewards for ASs in a complex network topology taking into account peering and transit accords, non trivial routes and different ASs sizes. Note that, once the model is developed, the ISPs could exploit this approach by tuning the parameters with accurate settings according to their knowledge and information availability.

In Figure 2 is depicted the topology used for experiments. The network is composed of eight different ASs, and nodes are connected with links characterized by peering or transit agreements. In the system there are $N = 10$ different resources types.

8.1. Changing AS sizes

We study the impact of changing the size of ASs. Different sizes are computed according to Zipf-Mandelbrot distribution with parameters $N = [1 \ldots 8]$ with step 1, $s = [0 \ldots 1.4]$ with a step of 0.2, and $q = 2$. Figure 9 shows how network sizes change. When $s = 0$ all the ASs have the same number of nodes, while moving on higher value of the s parameter the distribution becomes more skewed. First we use the uniform strategies for resources finding. If we set the cost usage to 1 for all links representing transit agreements from AS_i to AS_j($\sigma_{ij} = 1$), the rewards (Figure 10) and the costs (Figure 11) can be computed for each AS. It is interesting to use such information to compute *the net profit*, that is the difference between rewards and costs (shown in Figure 12), and to study how the profit changes as a function of the different AS size. We observe that in this particular topology AS_1, AS_2, and AS_7 only pay for transit access or have peering agreements, so their net profit is negative. For AS_1 and AS_2 the costs diminish when the value of the parameter s increases due to the reduced peer population. Instead for AS_7 the population increases with the value of s, so the transit fee it pays to AS_8 has a greater impact on its profit. AS_8 always increases its profit, taking advantage both from its position in the topology (i.e. since it has only peer agreements), and from the fact that it has the largest population that becomes even wider when the distribution is more skewed. Note that for AS_5 there is a trend inversion when $s = 0.6$. We can conclude that the more skewed peer distribution has a positive effect on AS from 1 to 3 (the one with a smaller population) and 8 (the one with the largest population), whereas is negative for 4 to 7 (the one with an average population).

8.2. Changing weights of the links

We use the weighted strategy to increase the weights on peering links. This experiment shows how the proposed methodology can be exploited to study the impact on costs and rewards among different ASs of an appropriate applicative routing protocol. In particular, we assigned $wgt_{ij} = 1$ for transit agreements and $wgt_{ij} = 1 + offset$ for local search ($i = j$) and peer agreements to increase the priority towards less expensive routes. Again we set the cost usage $\sigma_{ij} = 1$ for transit agreements from AS_i to AS_j. The total profit for each AS, when all ASs increase weights to their peering links, is illustrated in Figure 13 with $offset$ varying from 0 to 1.2. The absolute value of the fees paid in the overall system globally decreases. AS_1, AS_2, and AS_3 pay less than using the uniform strategy ($offset = 0$) and AS_8 has a lower profit.

We then study the effects that changes in the routing strategy of a single AS can cause to the entire system. Figure 14 shows the results when only AS_7 increases the routing weights on its peering links. As we can see, the changed policy of AS_7 affects the overall system. Results are obtained by changing the weights (of $AS7$ peering links) by setting $offsets$ from 0 to 6. Indeed for increasing value of wgt the total rewards of AS_7 increase, while the gains of AS_5, AS_6 and AS_8 diminish. AS_1, AS_2, AS_3 and AS_4 benefit from the new policy of AS_7, that uses less their transit links preferring its peering partners.

8.3. Changing costs of the links

In this experiment we change the cost for the links connected to AS_8, i.e. σ_{i8} for each $i \in (4, 5, 6)$, the value ranges from $\sigma_{i8} = 1.1$ to $\sigma_{i8} = 1.4$. The results are shown in Figure 15. Higher costs affect only the directly connected ASs. An increment of cost in the transit to AS_8 increases as expected the revenue of this AS at the expense of its neighbors. It is interesting however noting that other ASs in the system are not affected by this cost increase.

8.4. Changing AS agreements

Finally we consider changes to the agreements among ASs. The results are reported in Figure 16. We denote with M_1 the topology used in previous experiments and depicted in Figure 2 and with M_2 the one shown in Figure 3 obtained by changing the agreements in the following way: the links between AS_1 and AS_4, and from AS_2 to AS_4 have now a peering agreement; from AS_5 to AS_7 a transit agreement is settled where AS_5 is the customer. A similar connection is set up between AS_7 and AS_6, with AS_7 as customer. We illustrated results in Figure 16, considering the weighted strategy with two different weights, $wgt_{ij} = 1$ and $wgt_{ij} = 10$. In this case if we increase the weights to peering ASs we obtain a gain for all the networks. Also in this topology AS_1, AS_2, and AS_3 do not benefit from transit agreements, but AS_4 and AS_5 have now a positive profit because they do not route requests of AS_2. It is interesting to note that in the considered case the reduction of the number of transit agreements is more convenient (despite the reduction in profits) since the effect of reducing the expenses is predominant.

9. Conclusions

In this paper we have proposed a simple technique to study the economy relations, in terms of costs and revenues, among different ASs. The real effects of peering agreements depends on the traffic exchanged among the different ASs: a model of such traffic is thus required to better understand the effects of the considered agreements. We have thus presented a detailed model of the traffic generated by a simple P2P file sharing protocol.

The aim of this work is to provide a tool to help administrators in determining the right costs in commercial agreements, and to tune the system parameters to increase the profits and reduce the expenses system-wide. The case study proposed in this paper considers different resource location policies, and in particular the impact of being able to locate requested resources in the same AS has been emphasized.

The main limitations of this approach lies in the difficulty to get the required global information, since ASs are administrated by different entities. However, we showed a way to study the impact of parameters characterizing the application on the global topology. Future research directions should address techniques able to retrieve the required information and to automatically tune the applications.

Both the AS cost and reward determination model, and the P2P models, can be extended and used in different contexts. The former can be used to study the peering agreements among ASs under the traffic generated by different type of applications, and the latter can be exploited to study other effects of the traffic generated among peers in a more general overlay network.

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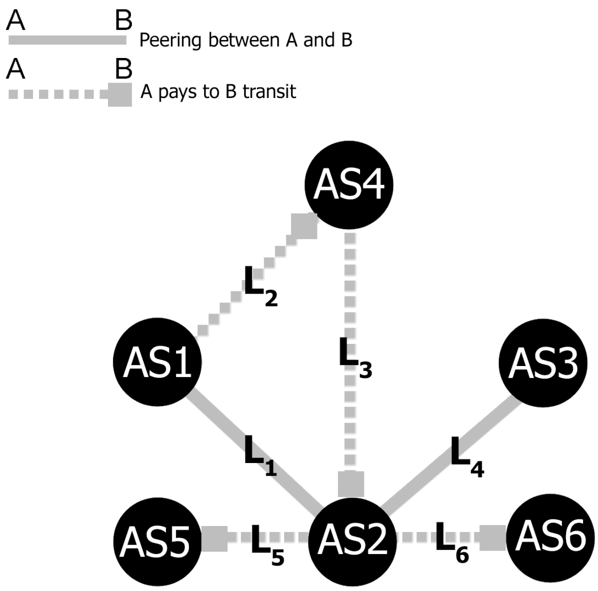

$$\mathbf{C^2} = \begin{bmatrix} 0 & 0 & 0 & 0 & \sigma(2,5) & \sigma(2,6) \\ 0 & 0 & 0 & 0 & \sigma(2,5) & \sigma(2,6) \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma(2,5) & \sigma(2,6) \\ \sigma(2,5) & \sigma(2,5) & 0 & \sigma(2,5) & 0 & 0 \\ \sigma(2,6) & \sigma(2,6) & 0 & \sigma(2,6) & 0 & 0 \end{bmatrix}$$

$$\mathbf{R^2} = \begin{bmatrix} 0 & 0 & \sigma(4,2) & 0 & \sigma(4,2) & \sigma(4,2) \\ 0 & 0 & 0 & \sigma(4,2) & 0 & 0 \\ \sigma(4,2) & 0 & 0 & \sigma(4,2) & 0 & 0 \\ 0 & \sigma(4,2) & \sigma(4,2) & 0 & \sigma(4,2) & \sigma(4,2) \\ \sigma(4,2) & 0 & 0 & \sigma(4,2) & 0 & 0 \\ \sigma(4,2) & 0 & 0 & \sigma(4,2) & 0 & 0 \end{bmatrix}.$$

Figure 1: Example topology of six ASs with the cost and reward matrices for AS_2

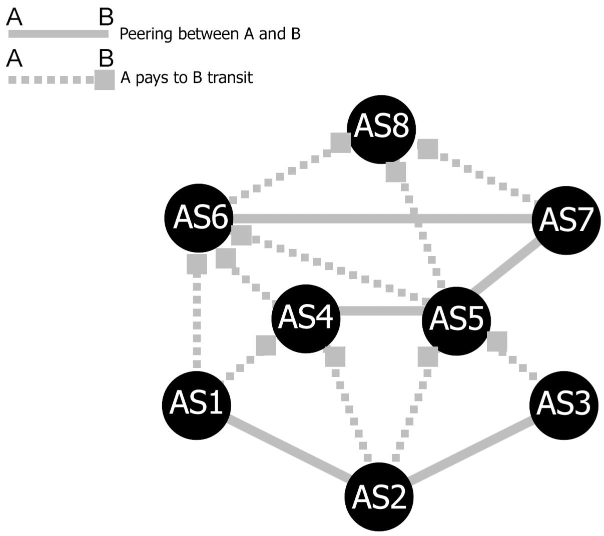

Figure 2: AS topology M_1 used for experiments

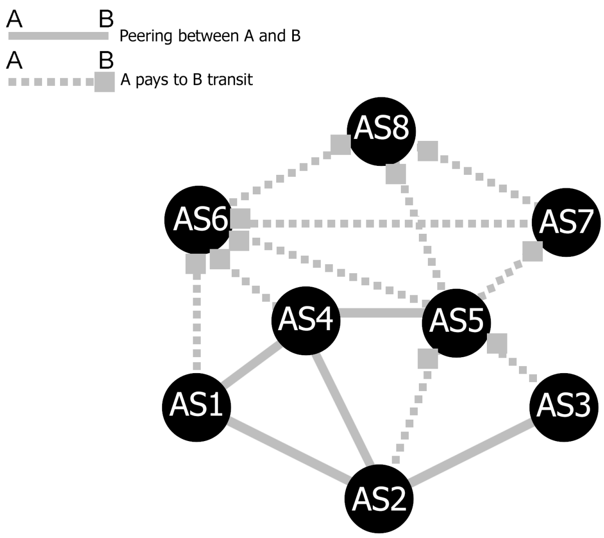

Figure 3: AS topology M_2 used for experiments

Figure 4: AS topology used to validate the model

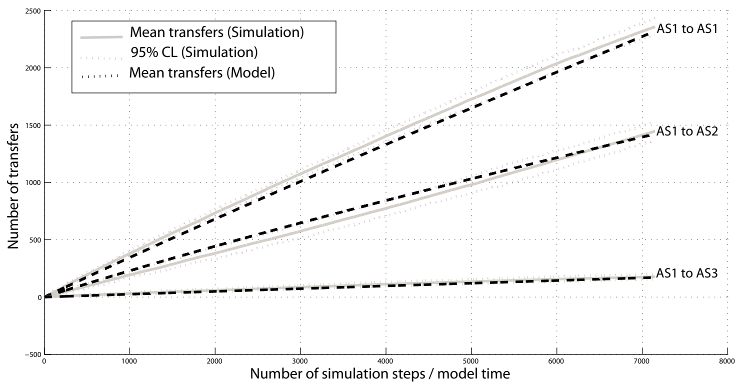

Figure 5: Comparison between the model and the simulation for transfers from AS_1 to others

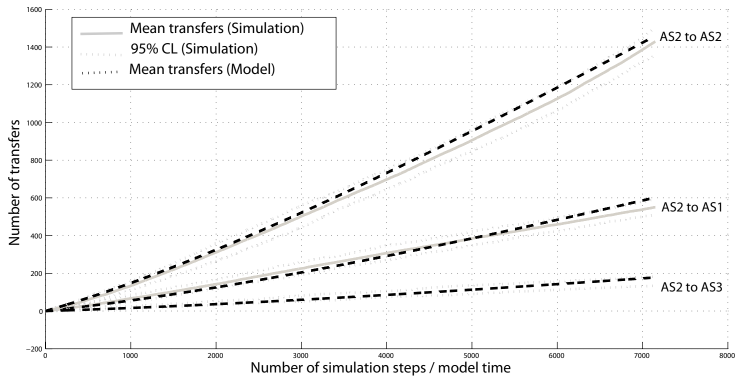

Figure 6: Comparison between the model and the simulation for transfers from AS_2 to others

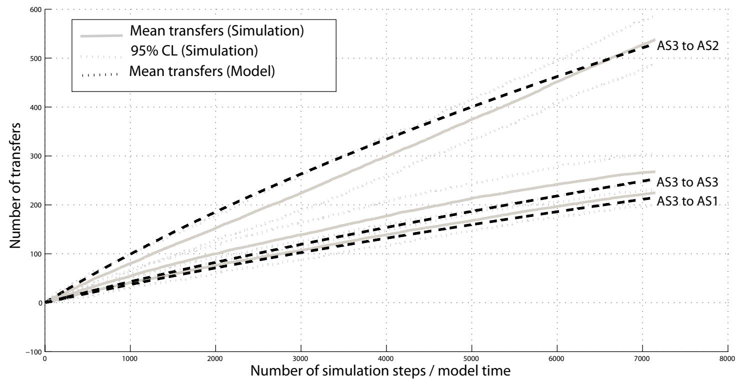

Figure 7: Comparison between the model and the simulation for transfers from AS_3 to others

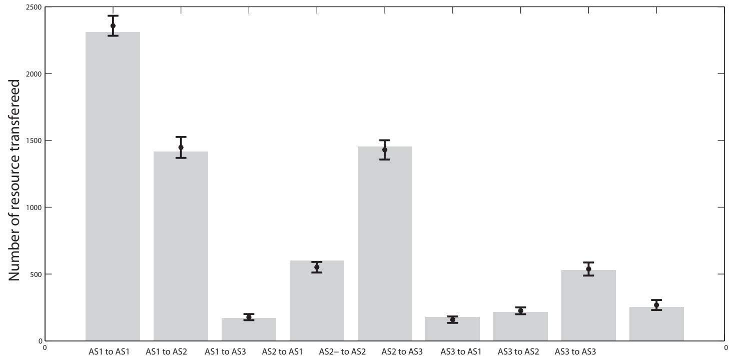

Figure 8: comparison between the model and the simulation in the steady state, the bars are the model outcome compared with the mean of 50 simulation runs and 95% confidence interval (dots with bars)

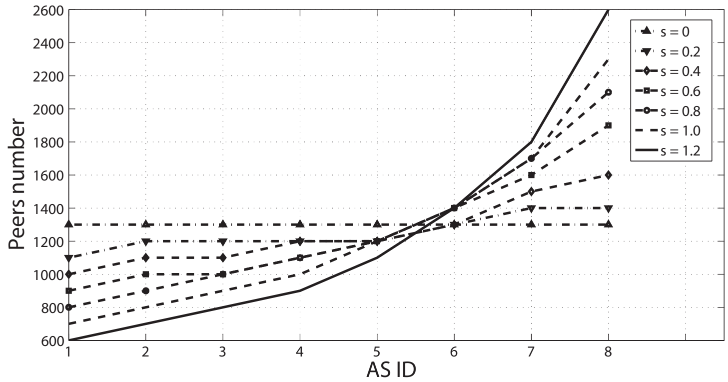

Figure 9: Different configuration of AS sizes

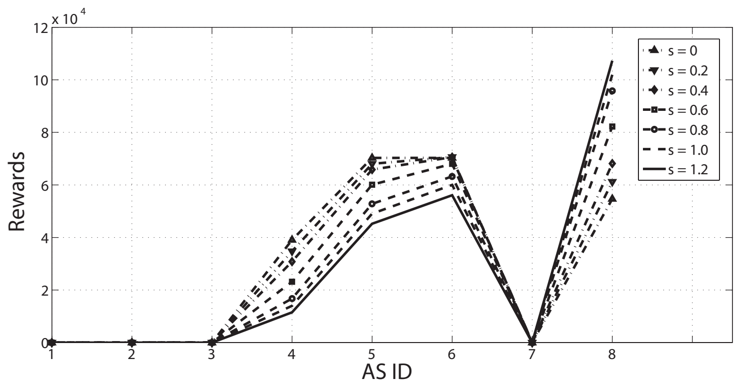

Figure 10: Reward distribution considering several configuration of AS sizes

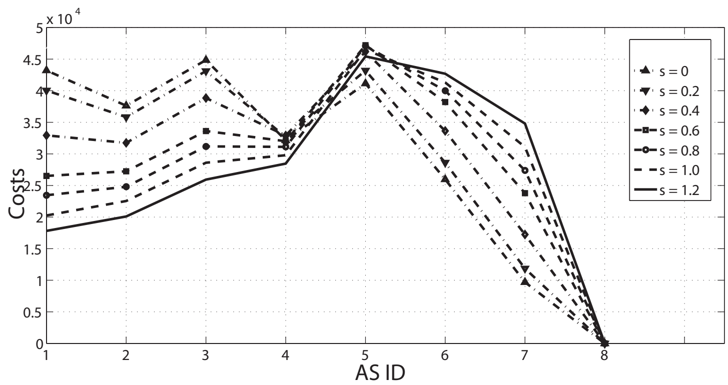

Figure 11: Cost distribution considering several configuration of AS sizes

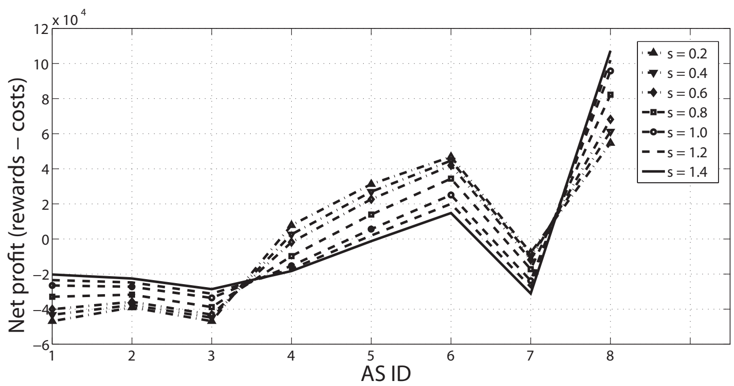

Figure 12: Distribution of the difference reward-cost (net profit) considering several configuration of AS sizes

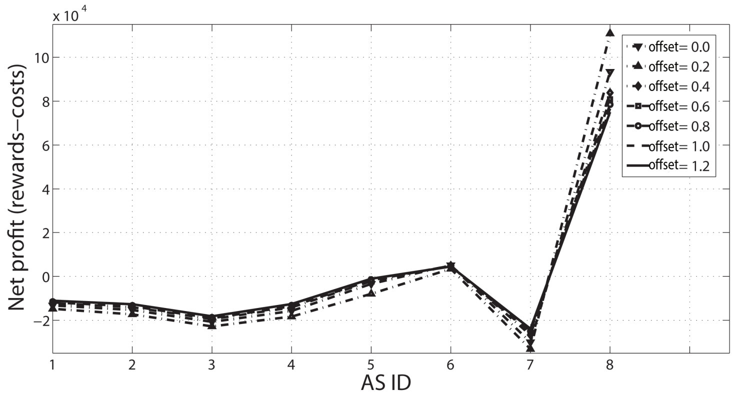

Figure 13: Distribution of the difference reward-cost (net profit) when all ASs increase weights to their peering links

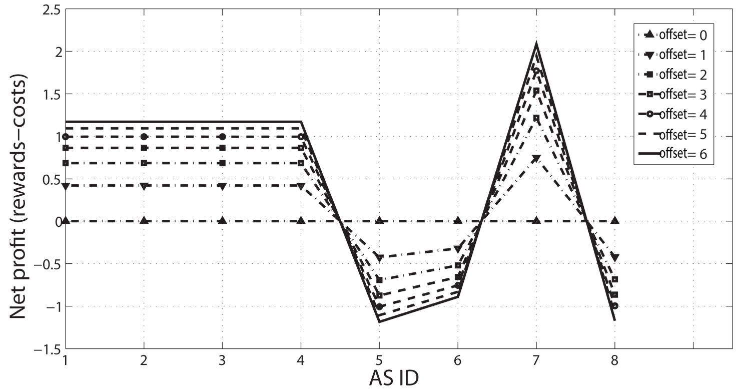

Figure 14: Distribution of the difference of the net profit respect to $wgt = 0$ when only AS_7 increases weights to its peering links

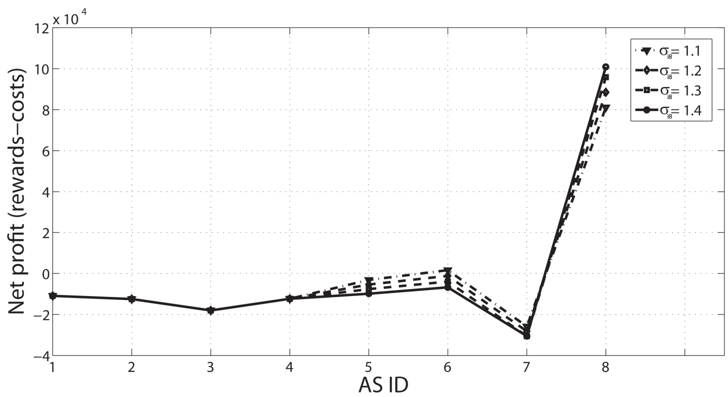

Figure 15: Net profit when only AS_8 increases its transit costs

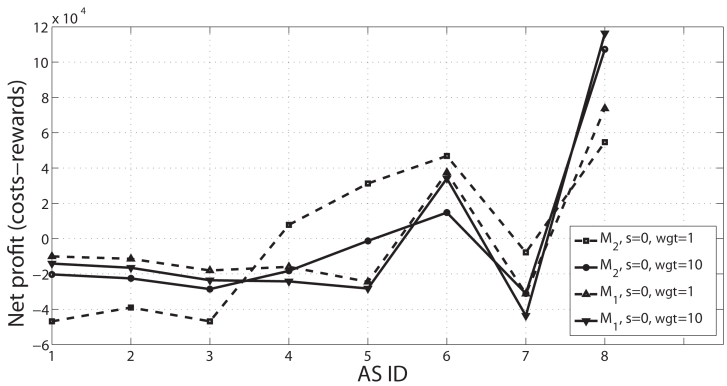

Figure 16: Comparison between two different topologies